taining an almost infinite information storage capacity—tends to lose bits of information in time.

Work is progressing at the FRSL to perform automatic feature discriminations using high-altitude and space-altitude photography. Initial efforts have been with single black-and-white transparencies from which 1000 separate density levels could be recorded. In the near future the addition of color recording filters will yield further analytical power to the film scanning device. The extent to which this type of approach to facilitating image analysis will be of great interest to land-use managers when the time comes for operational satellites to fly.

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# Determination of Radiation Properties of a Transparent Sheet Using Monte Carlo Method

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Recently the Monte Carlo Technique has been shown to be a powerful tool in the analysis of complex radiative heat transfer problems involving absorbing and emitting gases.[1] We have postulated a model for determining the radiation characteristics of dielectric body with a complex geometry. To substantiate this model, the theoretical radiation characteristics—absorptance, transmittance, and reflectance—were obtained by the Monte Carlo Method for a flat plate. These results were compared with Gardon's values obtained by a classical approach for emissivity.

The Monte Carlo Method is based upon following the probable path of a single energy bundle or particle history from initiation to termination.[2] Following this basic premise, consider the interaction of a single energy bundle at a fixed angle of incidence, with a sheet of dielectric material of uniform thermal and physical properties. The bundle of energy impinges on the top surface of the sheet at an angle of incidence $\eta$ with respect to a normal to the surface. The angle of transmission $\beta$ of the bundle of energy if it were transmitted into the sheet is given by Snell's law:

$$\sin\eta = n_{12} \sin\beta \tag{1}$$

where $n_{12}$ is the relative index of refraction of the surrounding medium and sheet, respectively. The external reflectivity of the interface is given by Fresnel's equation:

$$\rho_{\perp} = \sin^2(\eta - \beta)/\sin^2(\eta + \beta) \tag{2a}$$

and

$$\rho_{\parallel} = \tan^2(\eta - \beta)/\tan^2(\eta + \beta) \tag{2b}$$

where $\rho_{\perp}$ and $\rho_{\parallel}$ are the reflectivities for radiation polarized perpendicular and parallel to the plane of incidence, respectively. At this point in the particle history, a random number

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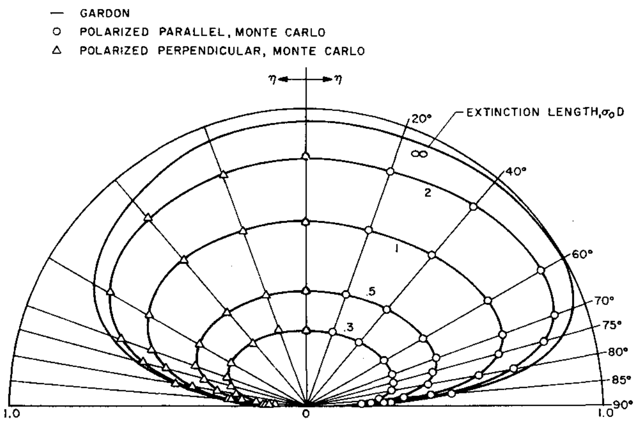


**Fig. 1 Absorptance of a sheet to perpendicular and parallel incident radiation.**

$R_n$ from a uniformly distributed set of numbers between zero and one is obtained. If the value of the random number obtained is less than or equal to the interfacial reflectivity, the particle is assumed to be reflected back into space and counted as such. If, however, the random number is greater than the interfacial reflectivity, the particle is assumed to be transmitted through the interfacial layer into the body at an angle $\beta$ with respect to a normal. The maximum path length the particle will travel within the sheet while in transit from the top to bottom surface is:

$$L = D/\cos\beta \tag{3}$$

However, there is the probability that the bundle of energy will be absorbed before it reaches the bottom surface. Howell and Perlmutter[1] have shown that the actual path length of the particle, giving the same distribution of path lengths as the fractional absorption distribution in a medium of uniform absorptivity $\tau$, is:

$$l = (1/\tau) \ln R_n \tag{4}$$

where $R_n$ is a different random number from a uniformly distributed set of numbers between zero and one. If the actual path length is equal to or less than the maximum path length, the energy bundle is absorbed within the sheet and counted as such. If, however, the actual path length $l$ is greater than the maximum path length $L$, the energy bundle interacts with the bottom interface. The bundle of energy can either be transmitted through the interface or reflected up towards the upper surface. Assuming the two surfaces of the sheet are parallel, the angle of incidence of the energy bundle on the bottom interface is $\beta$ and the angle of transmission will correspondingly be $\eta$. The reflectivities of the energy bundle

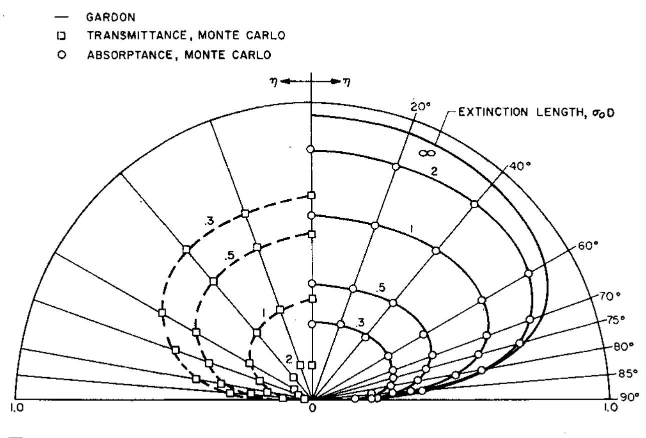


**Fig. 2 Absorptance and transmittance for diffuse radiation.**

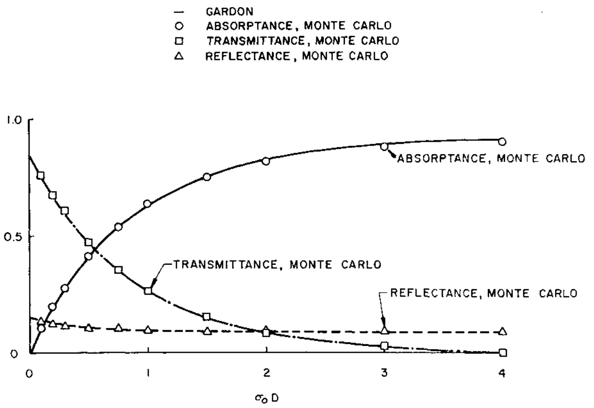


**Fig. 3 Radiation properties of transparent sheet to diffuse incident radiation.**

on the bottom interface as given by Eqs. (2a) and (2b), are identical to that on the upper interface since the absolute value of the sum and differences of the angles of incidence and transmission are identical for both surfaces. A third random number is picked to determine if the energy bundle is reflected from the bottom interface. If the particle is transmitted, it is counted as such. If, however, the particle is reflected, the angle of reflection will be equal to the angle of incidence $\beta$, from basic optics, and the maximum path length the particle can travel in reaching the top interface is again $L$. In reaching the top interface, point II, the particle will travel a total distance of $2L$. If the actual path length that the particle travels (originally greater than $L$) is now less than $2L$, the particle will now be absorbed. If, however, the particle reaches the top interface, it is determined whether the particle is reflected or transmitted into space. The process is repeated until the particle is either absorbed, reflected, or transmitted.

As each particle is followed, its ultimate destination is recorded. Since the final results are statistical in nature—that is, the number of particles which terminate in one position are compared to the total number of particles followed—it is necessary to determine how many histories are necessary to obtain meaningful results. Good convergence was obtained with 500 histories. However, for all the results presented in this paper, 10,000 histories per data point were used. The angular absorptance for both perpendicular and parallel polarized radiation are shown in Fig. 1. These results were compared to the analytical results for emissivity obtained by Gardon[3] for a semitransparent sheet. The angular absorptance and transmittance for diffuse radiation, which are numerically equal to one-half of the sum of their respective polarized components, are shown in Fig. 2.

To obtain the radiation characteristics of a semitransparent sheet to diffuse incident radiation, the preceding analysis has to be modified. Firstly, the angle of incidence $\eta$ of the energy bundle is not a constant but must be determined by a probabilistic method. Howell shows that for diffuse emission the angle of emission is given by

$$\sin\eta = R_n^{1/2} \qquad (5)$$

By direct analogy, the angle of incidence for diffuse irradiation is given by the above relationship. In addition, for each bundle of energy, the type or polarization, $\perp$ or $\|$, must be determined to calculate the interfacial reflectivities. These changes were incorporated into the previous computer program. The radiation characteristics—absorptance, transmittance, and reflectance—of a semitransparent sheet are shown in Fig. 3.

The complete radiation properties absorptance, emmitance, and transmittance of a semi-transparent sheet were determined by the Monte Carlo Technique. The absorptance values obtained by this analysis are in excellent agreement with the analytical results obtained by Gardon for emittance.

The authors have found that the adaptation of the Monte Carlo Technique as presented in this paper is one of the most powerful tools in the determination of the radiation properties of a semitransparent medium. Geometries for which an analytical solution is impossible to obtain can be analyzed by this technique.

## References


[1] Howell, J. R. and Perlmutter, M., "Monte Carlo Solution of Thermal Transfer Through Radiant Media Between Grey Walls," *Transactions of the ASME, Ser. C: Journal of Heat Transfer*, Vol. 86, 1964, pp. 116–122.

[2] Howell, J. R. and Siegel, R., *Thermal Radiation Heat Transfer*, Vol. 2, SP-164, NASA, 1969.

[3] Gardon, R., "The Emissivity of Transparent Materials," *Journal of the American Ceramic Society*, Vol. 39, No. 8, 1956, pp. 278–287.


# Swirling Flow through Multiple Nozzles


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## Nomenclature

$A, A^*$ = cross-sectional area and throat value
$A^*_{eff}$ = effective throat flow area
$c_f$ = thrust coefficient $\equiv F/P_cA^*$
$c^*$ = characteristic exhaust velocity
$D$ = diameter
$F$ = thrust
$J$ = $A^*/A_p$
$\dot{m}$ = mass flow rate
$P$ = pressure
$P_c$ = stagnation pressure at nozzle entrance
$S$ = nozzle center displacement from chamber centerline

### *Subscripts*

$a$ = atmospheric condition
$e,p$ = nozzle exit and grain port, respectively
$z$ = condition with radial injection and maximum $A_p$

## Introduction

For swirling flow in a nozzle, conservation of angular momentum requires that the swirl become more intense as the flow area decreases during passage of the particles through the convergent section of the nozzle. The intensity reaches a maximum at the throat, decreasing effective throat area, hence mass flow and thrust. Swirling discharge through a single nozzle has been treated theoretically,[1,5] and Batson and Sforzini[6] have obtained experimental confirmation of the fluid vortex structure in a choked nozzle. Lewellen et al.[4] have demonstrated that the flow upstream of the choked section is independent of downstream boundary conditions as is the case for nonswirling gas dynamics. Treatment of swirl-


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